Interval Notation

INTERVALS

Let a and b be two real numbers with a < b. Then,

A **closed interval** represented by \([a, b]\) consists of all real numbers \(x\) such that \(a \leq x \leq b\).

An **open interval** represented by \((a, b)\) consists of all real numbers \(x\) such that \(a < x < b\).

A **half-open, or half-closed** interval is \((a, b]\) consisting of all real numbers \(x\) such that \(a < x \leq b\), and, \([a, b)\) consisting of all real numbers \(x\) such that \(a \leq x < b\).

In the above definitions, \(a\) is the **left endpoint** and \(b\) is the **right endpoint** of the interval.

The symbol \(\infty\) is not a real number but notation used to indicate that there is no boundary in the positive direction. The symbol \(-\infty\) also is not a real number but notation used to indicate that there is no boundary in the negative direction. Watch how each is used in an interval.

\([a, \infty)\) Consists of all real numbers such that \(x \geq a\).

\((a, \infty)\) Consists of all real numbers such that \(x > a\).

\((-\infty, a)\) Consists of all real numbers such that \(x < a\).

\((-\infty, a]\) Consists of all real numbers such that \(x \leq a\).

\((-\infty, \infty)\) Consists of all real numbers!

**Examples**

1 < \(x\) < 3 would have the interval notation \((1, 3)\).

2 \(\leq\) \(x\) < 5 would have the interval notation \([2, 5)\).

\(x\) > 7 would have the interval notation \((7, \infty)\).

\(x\) \(\leq\) 5 would have the interval notation \((-\infty, 5]\).
Study the examples below on how graphs, algebra notation, and interval notation relate.

Using interval notation to write solutions to inequalities.

Solve each of the following inequalities, graph the solution, and name using interval notation.

1) \(4x + 3 > 2x - 11\) subtract 2x from both sides
   \(2x + 3 > -11\) subtract 3 from both sides
   \(2x > -14\) divide both sides by 2
   \(x > -7\)
   \((-7, \infty)\)

2) \(-8 < 3x - 2 < 4\) add 2 to all three parts
   \(-6 < 3x < 6\) divide by 3
   \(-2 < x < 2\)
   \((-2, 2)\)
Using interval notation to name the domain of functions.

Name the domain for each function below.

1) \( f(x) = x^2 + 3x - 5 \)

The domain of all polynomial functions is all real numbers. Since this is a polynomial function, the domain is \((-\infty, \infty)\).

2) \( g(x) = \frac{x - 1}{x + 3} \)

This is a rational function. The domain of a rational function is all real numbers except the values for \( x \) where the denominator is zero. Therefore, \( x \neq -3 \). The domain is \((-\infty, -3) \cup (-3, \infty)\). The \( \cup \) means or. The notation represents all numbers less than -3 are in the domain as well as all numbers greater than 3 are in the domain.

3) \( f(x) = \sqrt{x - 2} \)

The square root function is defined for real numbers only when the radicand (number under the radical) is nonnegative. Therefore, \( x - 2 \geq 0 \). Solving this inequality makes the domain \( x \geq 2 \). In interval notation, \([2, \infty)\).

4) \( g(x) = \frac{x}{x^2 - 4} \)

Again, this is a rational function. Factoring the denominator into \((x - 2)(x + 2)\) shows us that \( x \neq 2, -2 \). Both these values would make the denominator zero. The domain would be: \((-\infty, -2) \cup (-2, 2) \cup (2, \infty)\). This shows that all real numbers less than -2, the real numbers between -2 and 2, and the real numbers greater than 2 are all in the domain.

5) \( f(x) = \frac{1}{\sqrt{x - 5}} \)

The square root function now appears in the denominator and the denominator cannot be zero. This changes the domain of the square root function to just positive real numbers because the denominator cannot be zero. Therefore, \( x - 5 > 0 \). Solving the inequality yields \( x > 5 \). In interval notation, \((5, \infty)\).